TRINITY COLLEGE

IN NOMINE DOMINI

**YEAR 11**
**MATHEMATICS**
**SPECIALIST**

**Test 2, 2023**
**Calculator Allowed**
**Geometric Proofs & Vectors II**

**STUDENT'S NAME:** MARKING KEY [KRISZYK]

**DATE:** Monday 8th May **TIME:** 50 minutes **MARKS:** 50
**ASSESSMENT %:** 10

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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**Question 1** (~~4~~ 5 marks)

Determine, giving answers to one decimal place,

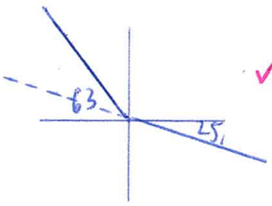
(a) the vector projection of $\underset{u}{12\underset{\sim}{i}+37\underset{\sim}{j}}$ onto $\underset{v}{75\underset{\sim}{i}-94\underset{\sim}{j}}$ (2 marks)

$$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = (\underset{\sim}{u}\cdot\hat{\underset{\sim}{v}})\times\hat{\underset{\sim}{v}}$$

$$= \left[\begin{pmatrix}12\\37\end{pmatrix}\cdot\frac{1}{\sqrt{75^2+94^2}}\begin{pmatrix}75\\-94\end{pmatrix}\right]\times\frac{1}{\sqrt{75^2+94^2}}\begin{pmatrix}75\\-94\end{pmatrix}$$

$$= \begin{pmatrix}-13.4\\16.8\end{pmatrix}$$

(b) the vector projection of a force of 60 N on bearing $333^\circ$ onto a force of 30 N on a bearing of $115^\circ$ (~~2~~ 3 marks)




$$\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = |\underset{\sim}{a}|\cos\theta\,\hat{\underset{\sim}{b}}$$

$$= 60\cos 142^\circ$$

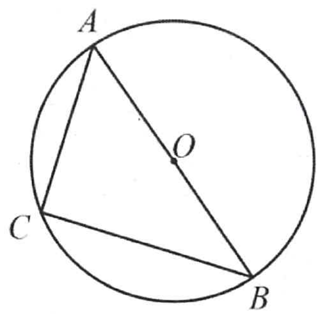
$= 47.3$ N ✓ on bearing

$180 + 115^\circ = 295^\circ$ ✓

accept $\begin{pmatrix}-42.868\\19.990\end{pmatrix}$

**Question 2** **(8 marks)**

(a) Determine, with justification, the length of the radius in the circle shown below given that $AC = 8$ cm and $BC = 15$ cm (2 marks)



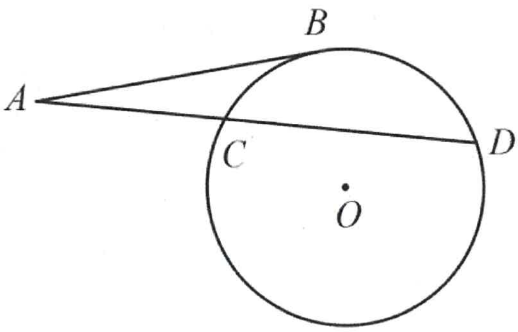

$\Delta ACB$ is right-angled at $C$

$\therefore \; d^2 = \sqrt{8^2+15^2}$

$= 17$

$\therefore \; r = 8.5$ cm

(b) Determine the length of the chord $CD$ given that the length of the tangent $AB$ is 15 cm and the length of the secant $AD$ is 26 cm. (3 marks)

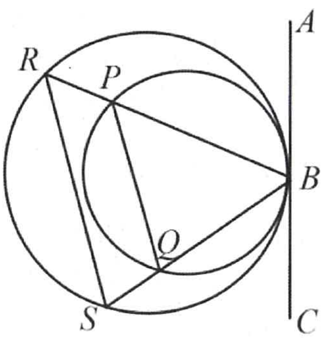



$AB^2 = AC \times AD$

$15^2 = (26-x) \times 26$

$x = 17.35$ cm

(c) The line segment $ABC$ is a common tangent to both circles shown below. Prove that $PQ$ is parallel to $RS$. (3 marks)




$\angle BPQ = \angle CBQ$ (alt. segment)

$\angle BRS = \angle CBQ$ (alt. segment)

$\therefore \angle BPQ = \angle BRS$

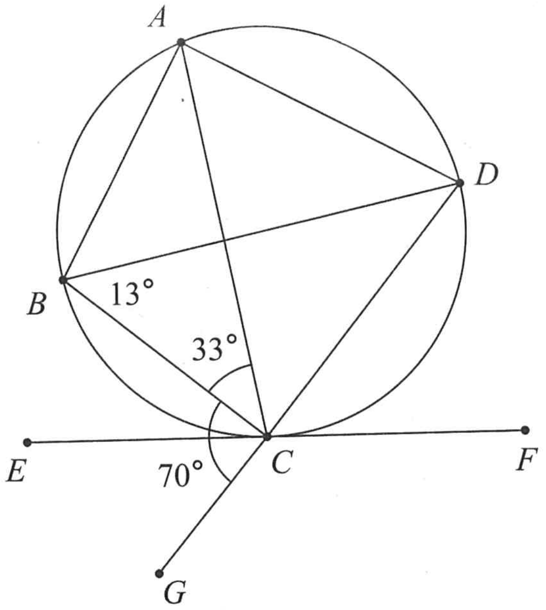
$\Rightarrow PQ \parallel RS$ (corresponding angles)

**Question 3** **(8 marks)**

$ABCD$ is a cyclic quadrilateral.

$EF$ is a tangent at $C$, and $DCG$ is a straight line.

$\angle BCA = 33°$
$\angle GCB = 70°$
$\angle DBC = 13°$




(a) Determine the following angles, giving reasons

(i) $\angle BAD$ (2 marks)

$\angle BCD = 110°$ (straight line)

$\angle BAD = 70°$ (opp $\angle$ in cyclic quad)

(ii) $\angle BDA$ (2 marks)

$\angle BDA = 33°$ (subtends same arc $\angle BCA$)

(b) Prove that $AC$ passes through the centre of the circle, justifying your answer. (4 marks)

$\angle ACD = 180 - 33 - 70$ (angles on line)
$= 77°$ ✓

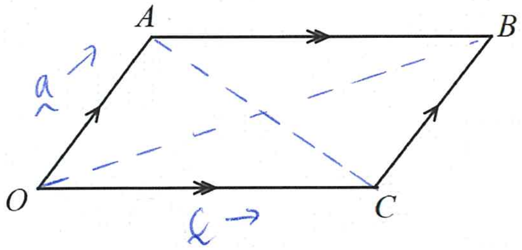
$\angle ABD = 77°$ (same arc) ✓

$\angle ABC = \angle ABD + \angle DBC$
$= 77° + 13°$
$= 90°$

As $\angle ABC$ subtends chord $AC$ and is $90°$, $AC$ must be the diameter. Q.E.D.

**Question 4** **(4 marks)**

Use vector methods to prove that the sum of the squares of the length of diagonals of parallelogram $OABC$ is equal to the sum of the squares of the length of the sides.




let $OA = \underset{\sim}{a}$

$OC = \underset{\sim}{c}$

$\therefore$ diagonals $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \underset{\sim}{a} + \underset{\sim}{c}$ ✓

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{a} + \underset{\sim}{c}$

RTP : $OA^2 + AB^2 + BC^2 + CO^2 = OB^2 + AC^2$

$|a|^2 + |c|^2 + |a|^2 + |c|^2 = |\underset{\sim}{a} + \underset{\sim}{c}|^2 + |-\underset{\sim}{a} + \underset{\sim}{c}|^2$

RHS $= |\underset{\sim}{a} + \underset{\sim}{c}|^2 + |-\underset{\sim}{a} + \underset{\sim}{c}|^2$

$= (\underset{\sim}{a} + \underset{\sim}{c}) \cdot (\underset{\sim}{a} + \underset{\sim}{c}) + (-\underset{\sim}{a} + \underset{\sim}{c}) \cdot (-\underset{\sim}{a} + \underset{\sim}{c})$ ✓

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{c} \cdot \underset{\sim}{c} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{c} \cdot \underset{\sim}{c}$ ✓

$= 2|a|^2 + 2|c|^2$ ✓

$=$ LHS

Q.E.D.

**Question 5** **(8 marks)**

(a) Consider the following true statement "if a hexagon is regular then it has six sides of equal length"

(i) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

If a hexagon does not have six sides of equal length then it is not regular. ✓

True - contrapositive. ✓

(ii) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

If a hexagon is not regular then it does not have six sides of equal length. ✓

False - angles may cause it to not be regular ✓

(iii) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

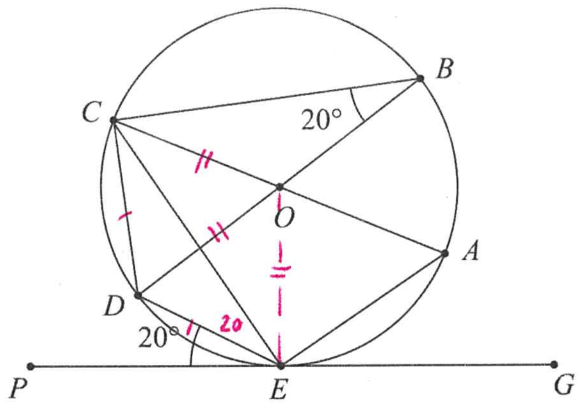
If a hexagon has six sides of equal length then it is regular. ✓

False - angles must also be equal for a regular polygon ✓

**Question 6** **(10 marks)**

In the diagram at right, $A$, $B$, $C$, $D$ and $E$ are five points on the circle with centre $O$.
$AC$ and $BD$ are diameters, and $PEG$ is a tangent to the circle at $E$.

It is given that $\angle DEP = \angle DBC = 20°$.




(a) Determine, giving reasons, the size of the following angles:

(i) $\angle DEC$ (1 mark)

$\angle DEC = 20°$ (angle subtends same arc)
$\angle DBC$

(ii) $\angle DCE$ (1 mark)

$\angle DCE = 20°$ (alt. segment)

(b) Using part (a) and triangle $CDE$, explain why $\angle COD = \angle DOE$. (2 marks)

$\Delta CDE$ is isosceles ✓

$\therefore$ $\angle COD = \angle DOE$ as equal chords subtend equal angles. ✓

(c) Prove that $\angle EAC = 40°$, giving reasons. (3 marks)

$\angle AEC = 90°$ (semi-circle)

$\angle AEG = 50°$ (straight line) ✓

$\angle ECA = 50°$ (alt segment to $\angle AEG$) ✓

$\angle EAC = 40°$ (Angle sum of $\Delta$ is $180°$) ✓

(d) Is it possible to draw a circle through the points $E$, $O$, $C$ and $D$. Justify your answer. (3 marks)

$\angle OCD = 50° + 20°$
$= 70°$ ✓

or

$\angle COE = 80°$
$\angle CDE = 140°$

$\angle OEA = 90° - 50°$
$= 40°$

$\angle OED = (90° - 40°) + 20°$
$= 70°$ ✓

As $\angle OCD + \angle OED \neq 180°$ points O, E, C, D cannot be a cyclic quadrilateral ✓

$\therefore$ circle cannot be drawn.

**Question 8** **(8 marks)**

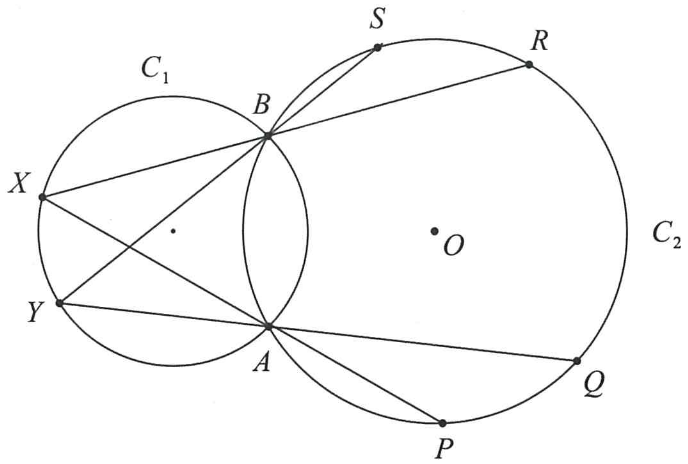
Two circles, $C_1$ and $C_2$, intersect at $A$ and $B$.

Two points $X$ and $Y$ are on $C_1$.

The line $XA$ extended intersects with $C_2$ at $P$, and $YB$ extended intersects with $C_2$ at $S$.

The line $XB$ extended intersects with $C_2$ at $R$, and $YA$ extended intersects with $C_2$ at $Q$.

$O$ is the centre of $C_2$.




(a) Prove that $\angle PAQ = \angle SBR$, giving reasons. (3 marks)

$\angle YAX = \angle YBX$ (subtend same arc) ✓

$\angle PAQ = \angle YAX$ (vertically opp.) ✓
$\angle SBR = \angle YBX$ ✓

$\therefore \angle PAQ = \angle SBR$

(b) Prove the chords $PR$ and $QS$ are congruent. (5 marks)

RTP $\angle POR = \angle QOS$ ✓

let $\angle PAQ = \angle SBR = \alpha$

$\therefore \angle POQ = 2\alpha$
$\angle SOR = 2\alpha$ (angles at centre are double) ✓

$\angle POR = \angle POQ + \angle QOR$

$\therefore \angle POR = 2\alpha + \angle QOR$ ✓

Chord PR subtends $\angle POR$

$\angle QOS = \angle SOR + \angle QOR$

$\therefore \angle QOS = 2\alpha + \angle QOR$ ✓

Chord QS subtends $\angle QOS$

$\therefore$ $PR = QS$ as they both subtend angle $2\alpha + \angle QOR$ at the centre. ✓

END OF QUESTIONS